Radiation Balance for a Room and a Sun Space: Collapsed Matrix Solution

Francisco Arumí-Noé, Ph.D.

ABSTRACT

A simple, yet correct and powerful, method of estimating the radiation performance of single spaces or of two coupled spaces is developed. This method is particularly useful for simplified numerical simulations of average lighting conditions and of radiant contribution to the heat balance of buildings. The article develops an example of the method and analysis of a room with an attached sunspace. The method is based on the properties of form factors. The form factors of two spaces coupled by radiation exchange are reduced to the form of a collapsed matrix. The form factors for each space are reduced to a 2×2 matrix subject to the principle of energy conservation. Radiation from one space is reflected from the surfaces and transmitted from one space to the other repeatedly in an infinite series. The result of the infinite series is obtained in explicit form for both the radiosity and the distributed form factor matrices. A parametric study of the interaction of a room with an attached sunspace is carried out explicitly. The results show a very weak dependence of the room absorptance on the surface reflectance and a very strong dependence of the illumination of the room on the same absorptance.

INTRODUCTION

The thermal behavior of a room attached to a sunspace depends on the distribution of the solar radiation among all surfaces. This distribution is primarily a geometry problem. For this article we adopt the geometry of the room and sunspace used by Wall (1997) where he compares predictions made by various simulation programs. In his conclusion, Wall lends greater credence to the results from TRNSYS/SUNREP (Bryn and Schiefloe 1996) and DEROB-LTH (Arumí-Noé 1979) because they "have appropriate methods to calculate solar gains" (Wall 1997). By "appropriate" he means that these simulation programs account for the geometry and the infinite series of reflections from the surfaces and transmissions from the room to the sunspace and back. The DEROB solution, first published in Arumí-Noé (1977), makes explicit use of the principle of energy conservation through the closure condition of form factors. Wall makes reference to the other programs used in his comparative study (Wall 1997) as using simplified methods that do not account properly for the geometric effects. He concludes that "simplified methods used in the design stage are appropriate" provided these methods include correct methods to calculate the distribution of the solar radiation. In this article I propose such a method.

The method is identical in form to the one developed for DEROB in that the closure and symmetry conditions for the form factors are arranged in matrix form, and, through a series of identity arguments, the closure condition is expressed in a different form so as to account for the infinite series of diffused reflections. The argument is quite general and independent of the number of surfaces used to describe each space. The information content of the solution increases as the surface detail increases so that the simulation matches more closely the "real" spaces. The computation cost also increases, except that it increases faster than the increase in information content. For many applications, especially early in the design stage, it is not necessary to have so much information, while it is very desirable to have an inexpensive tool. The method presented in this paper uses two surfaces to describe each space: one is a test surface and the other is the collected remaining surfaces. Thus, the matrices are collapsed to a 2×2 for each space. The values of the collapsed form factors are obtained directly from the closure and symmetry conditions without having to go through a long integration.

Francisco Arumí-Noé is a professor in the School of Architecture at the University of Texas at Austin.

There is only indirect empirical validation of the predictions of this simplified calculation. In fact, the validation is doubly indirect because the comparisons are made with the results of the calculations made with DEROB (Wall 1997). DEROB, in turn, has been validated extensively against empirical data collected for a number of existing buildings (Northrup and Arumí-Noé 1979, 1980; Wysocki and Arumí-Noé 1980; Arumí-Noé 1983) as well as for test structures (Arumí-Noé and Burch 1984). The experiments, however, were designed to test the global performance of buildings and not to isolate only the radiant exchange contribution.

This paper is organized in three sections. The first develops the collapsed matrix solution for two spaces coupled through the glazed partition they share. The method is applicable to any two spaces visually coupled, and, since the solution is for two spaces, the matrices are four-dimensional. The solutions are presented in closed form, which makes their implementation very fast and very inexpensive. The second section calculates the cavity absorptance and the albedo of the room and the attached sunspace. Cavity absorptance is the radiation absorbed by the surfaces enclosing the space and albedo is the radiation that entered the spaces and finds its way outdoors without being absorbed. The cavity absorptance is calculated for solar noon during the winter and summer solstices parametric on the surface absorptance. The third section compares the results from the method developed in this article with the results reported by Wall (1997). The article concludes with an appendix that contains the detailed results of the collapsed matrix solution.

FORM FACTORS

Review

The form factor G_{ij} is the fraction of the radiation from surface j that arrives to surface i on first incidence. It follows from energy conservation that the form factors among surfaces enclosing a space satisfy the closure condition:

$$\sum_{i=1}^{N} G_{ij} = 1 \tag{1a}$$

They also satisfy a symmetry condition:

$$G_{ij} A_j = G_{ji} A_i \tag{1b}$$

N is the number of separate surfaces used in describing the space. A_i is the area of surface i.

Collapsed Matrix

Concentrating our attention on one of the surfaces enclosing the space, we may call it the "reference surface." Collect the remaining surfaces enclosing the space into one effective convex surface. Label them as "1" and "2," respectively. The closure and symmetry conditions take the forms

$$\begin{cases} G_{11} + G_{21} = 1 \\ G_{12} + G_{22} = 1 \end{cases} \quad G_{12} A_2 = G_{21} A_1$$

When the reference surface is flat, $G_{11} = 0$, and then the closure and symmetry conditions are enough to determine the form factors to be

$$\begin{aligned} G_{21} &= 1 \\ G_{12} &= f \\ G_{22} &= 1 - f \end{aligned} \tag{2}$$

where

$$f \equiv \frac{A_1}{A_2}$$

All of the radiation from the reference surface is intercepted by the collected surfaces, while the radiation from the collected surfaces is partially intercepted by the reference surface and the rest goes to other parts of the collected surfaces. The proportion is given by the ratio of the areas. The form factors are collected as a matrix in such a way that the closure condition is written in explicit matrix form or in compact matrix notation:

$$\underline{\underline{G}} = \left(\begin{array}{c|c} 0 & f \\ \hline 1 & 1-f \end{array}\right) \quad (1, \quad 1)\begin{pmatrix} 0 & f \\ 1 & (1-f) \end{pmatrix} = (1, \quad 1)$$

$$\text{or } \underline{u} \cdot \underline{\underline{G}} = \underline{u} \tag{3}$$

where $\underline{u} = (1, 1)$ plays the role of a summation operator.

Two Connected Spaces

Consider two adjacent spaces, A and B, that share one glazed partition so that radiation from one space may enter the other. Define the collapse matrix for each space and select either side of the glazed partition as the reference surface for each respective space. A fraction of the radiation from surface 2 is transmitted through surface 1 and goes on to strike surface 4. Therefore, only part of the radiation that arrives on surface 1 is available to be either absorbed or reflected.

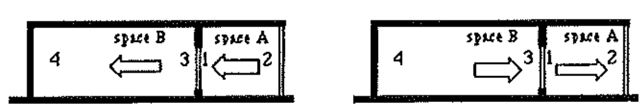

Figure 1 Radiation from space 1 enters space 2 and vice versa.

The same type of events occur to the radiation going from space B to space A. By energy conservation, the closure relation takes the form

$$(1\ 1\ 1\ 1)\begin{bmatrix} 0 & 1-t_1f_1 & 0 & 0 \\ 1 & 1-f_1 & 0 & t_3f_2 \\ 0 & 0 & 0 & (1-t_3)f_2 \\ 0 & t_1f_1 & 1 & 1-f_2 \end{bmatrix} = (1\ 1\ 1\ 1)$$

The transmittance coefficients, through either direction of the glazed partition, are t_1 and t_3. The elements of this matrix satisfy closure, but they do not satisfy the symmetry condition; therefore, they do not represent the form factors. Note, however, that by factoring the matrix,

$$(1\ 1\ 1\ 1)\begin{bmatrix} 1-t_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1-t_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & f_1 & 0 & 0 \\ 1 & 1-f_1 & 0 & t_3f_2 \\ 0 & 0 & 0 & f_2 \\ 0 & t_1f_1 & 1 & 1-f_2 \end{bmatrix} = (1\ 1\ 1\ 1)$$

we obtain the closure relation in terms of the form factor matrix and a modifier. In compact notation $\underline{u}\cdot\underline{\underline{\mu}}\cdot\underline{\underline{G}} = \underline{u}$ where

$$where G = \begin{bmatrix} 0 & f_1 & 0 & 0 \\ 1 & 1-f_1 & 0 & t_2f_2 \\ 0 & 0 & 0 & f_2 \\ 0 & t_1f_1 & 1 & 1-f_2 \end{bmatrix} f_1 \equiv \frac{A_1}{A_2} f_2 \equiv \frac{A_3}{A_4} \tag{4}$$

is the form factor matrix. The symmetry conditions are met provided $t_1 = t_3$. Since the absorptance, reflectance, and transmittance coefficients of each surface satisfy energy conservation, $a + r + t = 1$, the modifying matrix has the equivalent forms

$$\underline{\underline{\mu}} = \underline{\underline{a}} + \underline{\underline{r}} + \underline{\underline{\tau}} \tag{5a}$$

$$\underline{\underline{\mu}} = \begin{pmatrix} 1-t_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1-t_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a_1+r_1 & 0 & 0 & 0 \\ 0 & t_2+a_2+r_2 & 0 & 0 \\ 0 & 0 & a_3+r_3 & 0 \\ 0 & 0 & 0 & t_4+a_4+r_4 \end{pmatrix} \tag{5b}$$

The reflectance $\underline{\underline{r}}$, absorptance $\underline{\underline{a}}$, and the transmittance to the outdoors $\underline{\underline{\tau}}$ are

$$\underline{\underline{r}} = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix} \quad \underline{\underline{a}} = \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix} \quad \underline{\underline{\tau}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & t_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_4 \end{pmatrix} \tag{5c}$$

It is worth noting that the modifying matrix falls short of being the unit matrix because it does not include the transmittance coefficients of the interior surfaces. Therefore, for a single space, the modifying matrix is the unit matrix: $\underline{\underline{1}} = \underline{\underline{a}} + \underline{\underline{r}} + \underline{\underline{t}}$.

Distributed Form Factors

Manipulate the closure relation with a series of identity operations in order to obtain the *distributed* form factors. Distributed form factors include the infinite series of reflections that follow after the first incidence.

$$\underline{u}\cdot\underline{\underline{\mu}}\cdot\underline{\underline{G}} = \underline{u} \quad \Rightarrow \quad \underline{u}\cdot\left(\underline{\underline{a}}+\underline{\underline{\tau}}+\underline{\underline{r}}\right)\cdot\underline{\underline{G}} = \underline{u}$$

$$\Rightarrow \quad \underline{u}\cdot\left(\underline{\underline{a}}+\underline{\underline{\tau}}\right)\cdot\underline{\underline{G}} = \underline{u}\cdot\left(\underline{\underline{1}}-\underline{\underline{r}}\cdot\underline{\underline{G}}\right)$$

$$\Rightarrow \quad \underline{u}\cdot\left(\underline{\underline{a}}+\underline{\underline{\tau}}\right)\cdot\underline{\underline{G}}\cdot\left(\underline{\underline{1}}-\underline{\underline{r}}\cdot\underline{\underline{G}}\right)^{-1} = \underline{u}$$

$$\Rightarrow \quad \underline{u}\cdot\left(\underline{\underline{a}}+\underline{\underline{\tau}}\right)\cdot\underline{\underline{J}} = \underline{u}$$

The last expression is the closure relation for J_{ij}, the distributed form factors. J_{ij} is the fraction of the radiation from surface j arriving on surface i after all internal reflections. The elements of the distributed form factors are given by

$$\underline{\underline{J}} = \underline{\underline{G}}\cdot\underline{\underline{\sigma}} \tag{6}$$

where

$$\underline{\underline{\sigma}} = \left(\underline{\underline{1}}-\underline{\underline{r}}\cdot\underline{\underline{G}}\right)^{-1} \tag{7}$$

is the luminance matrix. Its form is a clear indication that it represents the infinite power series of the product $\underline{\underline{r}}\cdot\underline{\underline{G}}$. The element δ_{ij} is the contribution of surface j to the luminance of surface i. The explicit expression follows directly. The explicit form of the form factor matrix, $\underline{\underline{G}}$, for clear glass is given in the main text by Equation 4 and in Appendix A. This appendix also gives the form of this matrix for translucent glass. The surface property matrices, including $\underline{\underline{r}}$, are given in Equations 5c. Use these to carry out the operations indicated in Equation 7 to obtain the luminance matrix, $\underline{\underline{\sigma}}$. Appendix A gives the explicit form of this result. Then carry out the product indicated by Equation 6 and we obtain the distributed form factor matrix (see Equation 8).

Note that the distributed form factors obey the symmetry relation $J_{ij}A_j = J_{ji}A_i$.

The diffuse radiation received by surface i is given by

$$R_i \equiv \sum_j J_{ij} S_j$$

S_j is the primary radiant source from surface j; its value is independent of the matrix solution. R_i is the radiation received

$$\underline{\underline{J}} = \frac{1}{\delta}\left(\begin{array}{cc|cc} f_1 r_2 \delta_2 & f_1 \delta_2 & t r_2 r_4 f_1 f_2 & t r_2 f_1 f_2 \\ \delta_2 & [1 - f_1(1 - r_1)]\delta_2 + t^2 r_4 f_1 f_2 & t f_2 r_4 & t f_2 \\ \hline t r_2 r_4 f_1 f_2 & t r_4 f_1 f_2 & f_2 r_4 \delta_1 & f_2 \delta_1 \\ t f_1 r_2 & t f_1 & \delta_1 & [1 - f_2(1 - r_3)]\delta_1 + t^2 r_2 f_1 f_2 \end{array}\right) \tag{8}$$

by surface i; its value is dependent on the matrix solution. The radiation absorbed by surface i is $\alpha_i \equiv a_i R_i$, which is in proportion to the radiation received according to the absorptance coefficient of the surface. The radiation lost to the outdoors through surface i is $\psi_i \equiv \tau_i R_i$, which is in proportion to the radiation received according to the transmittance coefficient to the outdoors of the surface. We may collect the primary radiant source values in a column vector, whose with component is the primary source from surface i. Likewise, we may collect in column vectors the radiation received, absorbed, and lost to the outdoors:

$$\underline{R} = \underline{\underline{J}} \cdot \underline{S} \quad \underline{\alpha} = \underline{\underline{a}} \cdot \underline{R} \quad \underline{\psi} = \underline{\underline{\tau}} \cdot \underline{R} \tag{9}$$

The sum of the elements of $\underline{\alpha}$ yields the radiation absorbed by all surfaces. The sum of the elements of $\underline{\psi}$ yields the radiation lost to the outdoors through all of the surfaces. We expect from energy conservation that the sum of these two totals may be no more and no less than the total primary radiant source. We prove that this is indeed the case by using the closure relation for the distributed form factors:

$$\underline{u} \cdot (\underline{\alpha} + \underline{\psi}) = \underline{u} \cdot (\underline{\underline{a}} + \underline{\underline{\tau}}) \cdot \underline{\underline{J}} \cdot \underline{S} = \underline{u} \cdot \underline{S} \tag{10}$$

THE SUN SPACE

The Radiant Sources

A room is attached to a sunspace as shown in Figure 2. At solar noon, the solar beam enters the sunspace through the south glass and strikes the floor, back wall, and back window of the sunspace. From the geometry of the building, and from the solar angle, we may use geometric logic to find how much power that enters the sunspace strikes the various surfaces. The left image in Figure 3 illustrates the radiation incident on

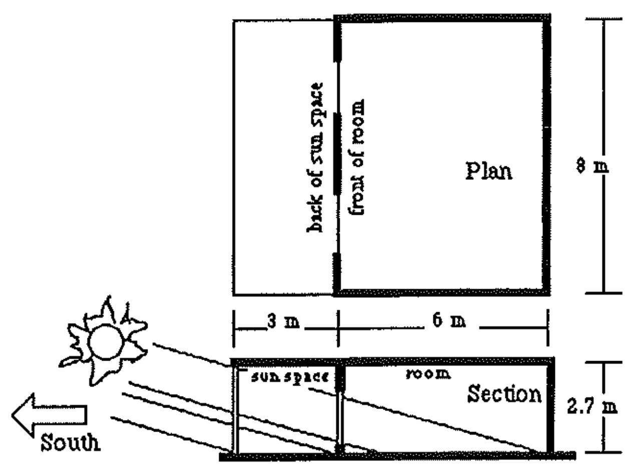

Figure 2 *Sun position and sun patches are for solar noon, December 21, latitude 55°N.*

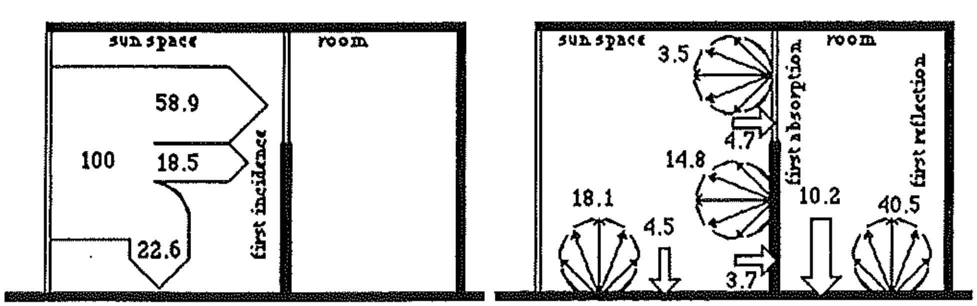

Figure 3 *Left: first incidence of solar radiation that enters the sunspace. Right: first absorption and first reflectance.*

TABLE 1 *

First Incidence	Incident	Absorbed	Reflected
Sun Space Floor	22.6	4.5	18.1
Sun Space Back Wall	18.5	3.7	14.8
Sun Space Back Glass	58.9	4.7	3.5
Room Floor		10.2	40.5
Total	100.0	23.1	76.9

* Distribution of First Incidence Radiation That Enters the Sun Space. Glass transmittance = 86%; reflectance = 6% for glass and 80% for opaque surfaces

each of the surfaces for every 100 units of power that enter the sunspace.

Each surface absorbs the radiation incident on it in proportion to its absorptance; likewise, each surface reflects the radiation incident on it in proportion to its reflectance. The respective values are illustrated in the right-hand image of Figure 3. The incident, absorbed, and reflected radiation are summarized in Table 1. The surface absorptance values are 20% for opaque surfaces and 6% for the glass. The glass has a transmittance of 86%. The sum of the radiation absorbed and reflected equals the radiation incident. The exception, of course, is for the back glass of the sunspace. The radiation that is neither absorbed nor reflected is transmitted through to the back room where it strikes the floor.

The absorbed value is the first contribution to the heat absorbed by the surface. The reflected radiation is assumed to be 100% diffuse. It is this value that is treated as the primary radiant source for the respective surface. It is the "S" value in Table 2.

The Distributed Form Factor Matrix

The distribution of these radiant sources is governed by the distributed form factors. We determine the coefficients associated with the reference and collected surfaces by tabulating the area, reflectance, transmittance, and radiant source associated with each surface.

The area, reflectance, transmittance, and radiant sources for the reference surfaces are the respective values on either side of the glazed partition shared by the spaces. The area and radiant source for the collected surfaces are what is left over from the respective totals. The reflectance and transmittance values for the collected surfaces are the respective averages weighted by area. Thus, the coefficient for the collapsed matrices have the values listed in Tables 2 and 3.

The matrices for the distributed form factors, the absorptance, and the transmittance to the exterior become

$$\underline{\underline{J}} = \left(\begin{array}{cc|cc} .12 & .23 & .02 & .03 \\ 1.84 & 1.67 & .37 & .46 \\ \hline .02 & .05 & .24 & .30 \\ .39 & .78 & 3.98 & 3.72 \end{array}\right) \quad \underline{\underline{a}} = \left(\begin{array}{cc|cc} .08 & 0 & 0 & 0 \\ 0 & .15 & 0 & 0 \\ \hline 0 & 0 & .08 & 0 \\ 0 & 0 & 0 & .20 \end{array}\right) \quad \underline{\underline{\tau}} = \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & .34 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

TABLE 2
Coefficient Values*

	A	r	rA	t	tA	S
Sun Space						
Floor	24	.80	19.20	0	0	18.1
Ceiling	24	.80	19.20	0	0	0
Glazed Sides	37.8	.06	2.27	.86	32.51	0
Back Opaque	9.6	.80	7.68	0	0	14.8
Back Glass	12.0	.06	.72	.86	10.32	3.5
Sum	107.4		49.07		42.83	36.4
Room						
Floor	48	.80	38.4	0	0	40.5
Ceiling	48	.80	38.4	0	0	0
Front Opaque	9.6	.80	7.68	0	0	0
Other Opaque	54	.80	43.2	0	0	0
Front Glass	12.0	.06	.72	.86	10.32	0
Sum	171.6		128.4		10.32	40.5

* All area terms are measured in m^2. The surfaces of the glazed partition are labeled as "back glass" and "front glass"; they are the reference surfaces for the sunspace and the attached room, respectively. The column labeled "S" is the primary diffused radiant source

TABLE 3
Parametric Description of Spaces

Sun Space	A_1	A_2	r_1	r_2	t_1	t_2	a_1	a_2	S_1	S_2	f_1	δ_1
	12	95.4	.06	.51	.86	.34	.08	.15	3.53	32.90	.13	.55
Room	A_3	A_4	r_3	r_4	t_3	t_4	a_3	a_4	S_3	S_4	f_2	δ_2
	12	159.6	.06	.80	.86	0	.08	.20	0	40.51	.08	.26

The Radiation Absorbed and the Radiation Lost

The primary radiation source, the radiation received, radiation absorbed, and lost radiation are summarized in the following column vectors:

$$\underline{S}=\begin{pmatrix}3.53\\32.90\\\hline 0\\40.51\end{pmatrix}\quad \underline{R}=\underline{\underline{J}}\cdot\underline{S}=\begin{pmatrix}9.25\\80.19\\\hline 13.74\\177.74\end{pmatrix}\quad \underline{\alpha}=\underline{\underline{a}}\cdot\underline{R}=\begin{pmatrix}.74\\12.23\\\hline 1.10\\35.55\end{pmatrix}\quad \underline{\psi}=\underline{\underline{\tau}}\cdot\underline{R}=\begin{pmatrix}0\\27.33\\\hline 0\\0\end{pmatrix}$$

For surface i, S_i is the radiant source, R_i is the radiation received after all reflections, α_i is the total radiation absorbed, and ψ_i is the total radiation transmitted to the outdoors. This information is listed by space in Table 4.

TABLE 4
Distribution of First Reflected Solar Radiation

	Absorbed	Lost	Source
Sun Space	$\alpha_1 + \alpha_2 = 12.9$	$\psi_1 + \psi_2 = 27.3$	$S_1 + S_2 = 36.4$
Room	$\alpha_3 + \alpha_4 = 36.7$	$\psi_3 + \psi_4 = 0$	$S_3 + S_4 = 40.5$
Total	49.6	27.3	76.9

As we expect from energy conservation, the total radiation absorbed and lost equals the total of all sources, and when we include the radiation that was absorbed on first incidence of the solar beam, we have the distribution listed in Table 5. And again, as we expect from energy conservation, the sum of all radiation lost to the outdoors and what is absorbed by both spaces equals the total radiation that entered the sunspace. The cavity absorptance of the sun space is 25.9% and of the attached room is 46.8%, and the albedo of the combined spaces is 27.3%. The radiation absorbed by the sunspace is twice what it was after the first incidence, and the radiation absorbed by the back room is more than quadrupled.

TABLE 5
Distribution of Solar Radiation in the Sunspace and Attached Room

	Absorbed After First Incidence	Absorbed After All Reflections	Total Absorbed	Lost to the Outdoors
Sun Space	12.9	13.0	25.9	27.3
Room	10.2	36.6	46.8	0
100.0			72.7	27.3

Summer Sun and Translucent Glass

At solar noon on June 21, the solar beam strikes only the floor of the sunspace. Twenty percent of it is absorbed, then the remaining 80% is reflected and acts as a radiant source. Only the floor of the sun space has a radiant source; therefore, the solar radiation that enters the attached room is the result of reflections from surfaces enclosing the sun space. Since much of it finds its way to the outdoors, the cavity absorptance of the attached room is considerably less than in wintertime (see Figure 4).

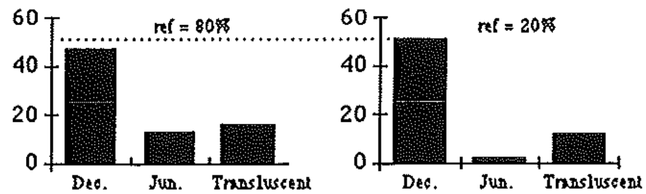

Figure 4 Cavity absorptance of the attached room.

Both winter and summer analyses assume that the glass enclosing the sunspace is clear. When this glass is translucent, no beam enters the sunspace. The only radiation that enters is diffused through the glass; thus, the exterior windows act as the radiant sources. In this case, the cavity absorptance of the attached room is independent of the time of the year. Note that we are making the distinction of clear or translucent glass only for the exterior glass of the sun space and not for the glass connecting the sunspace to the attached room.

When the surface reflectance decreases, a greater fraction of the first incidence radiation is absorbed; thus, there is less radiation available for distribution by reflection. In December the cavity absorptance of the back room is not very sensitive to the change in the surface reflectance, whereas in June it is very sensitive.

Parametric Analysis

We gain additional insight into the radiant behavior of these spaces when we vary the surface absorptance. Consider three variants: the surface absorptance of the sun space and of the attached room vary simultaneously, the absorptance of the surfaces of the sun space is held constant, and the absorptance of the surfaces of the attached space is held constant.

When all absorptance values vary simultaneously, the cavity absorptance of the attached room increases rapidly, at first with increasing surface absorptance. Then this rapid rise slows down, and eventually the cavity absorptance actually decreases with increasing surface absorptance (see Figure 5).

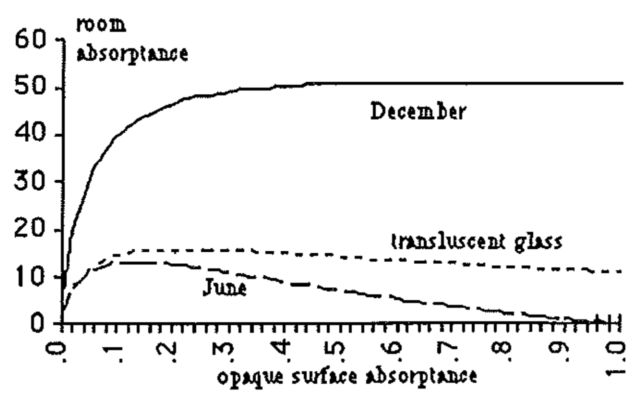

Figure 5 Cavity absorptance of attached room as a function of surface absorptance. All opaque surfaces have the same absorptance.

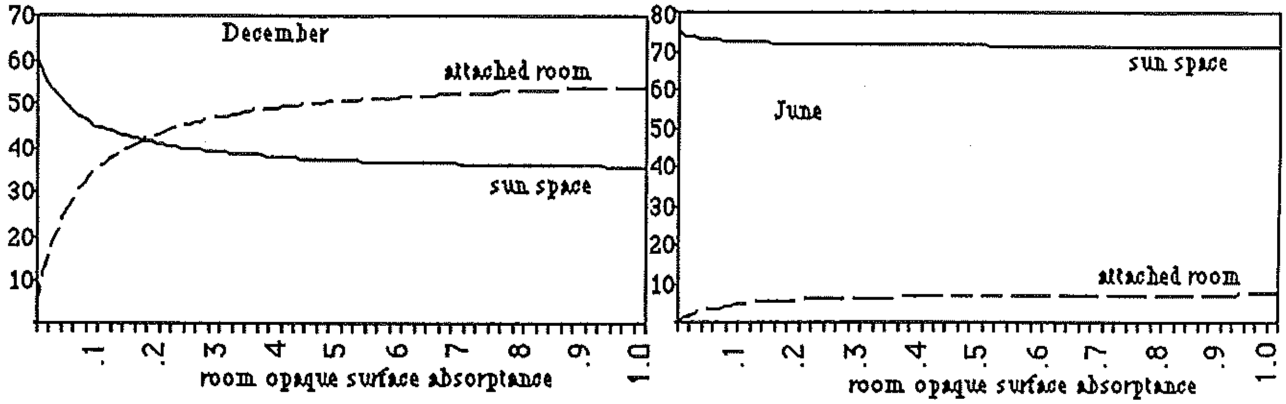

Figure 6 *Cavity absorptance, constant opaque surface reflectance (50%) of sun space.*

This turn around is most evident in June, when the only radiation that enters the attached room is due to the reflection of the beam from the floor of the space; so as the absorptance increases, there is less radiation available from reflections. This effect is not as dramatic but it is present for translucent glass. In that case, radiation enters the attached room from the diffusing glass before reflections redistribute the radiation. In December the cavity absorptance appears to level off toward a constant value. In fact, there is a slight decrease in the value toward the higher values of surface absorptance, but this decrease is so slight that we may ignore it.

When the surface absorptance of the sun space is held constant, the cavity absorptance of the attached room increases monotonically with increasing opaque absorptance of the room surfaces, very fast for low surface absorptance, and then it slows down for higher surface absorptance (see Figure 6). The cavity absorptance of the sunspace mirrors the behavior of the attached room. It decreases monotonically, very fast for low surface absorptance and progressively more slowly for higher surface absorptance. This slow, gradual change takes place over the range of surface absorptance most likely to be found with building materials. Under these conditions, the radiant behavior is weakly dependent on the surface absorptance. Therefore, the designer may choose almost any color that he or she wishes.

These conclusions are not as emphatic when we hold constant the surface absorptance of the attached room (see Figure 7). The cavity absorptance of the attached room decreases monotonically instead of increasing. The rate of change is faster for low absorptance, but the rate of change does not slow down as fast as in the other two cases. In fact, the cavity absorptance of the sunspace increases rapidly through the entire range of surface absorptance values.

For translucent glazing into the sunspace, we find the same pattern of behavior (see Figure 8).

Comparison with DEROB and TRNSYS

The room and the attached sun space used in the previous section were taken directly from the building used by Wall (1997), with the difference that the roof of the sunspace I used is opaque. Since Wall's sunspace has a glazed roof, I comform to his design for the purpose of comparison. In the previous section, all of the calculations were done for solar noon. The data Wall reports is for entire days, one in December and one in June. Wall used solar data from Copenhagen, Denmark. I used standard sea level data for latitude 55°.

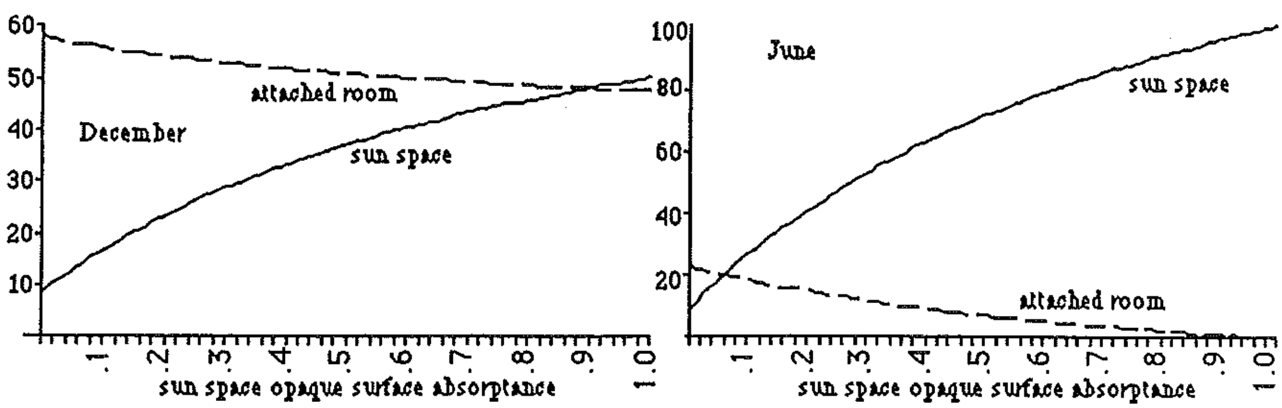

Figure 7 *Cavity absorptance, constant opaque surface reflectance (50%) of attached room.*

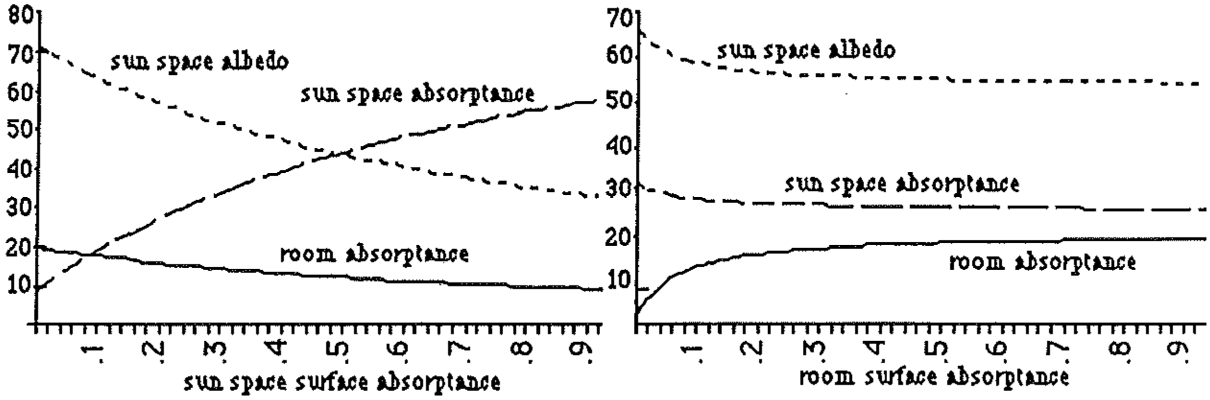

Figure 8 *Cavity absorptance and albedo. Sunspace exterior glazing is translucent. Left: Attached room has a constant surface absorptance. Right: Sunspace has a constant surface absorptance.*

Table 6 shows that all three simulations exhibit the same behavior. The radiation absorbed by the sunspace increases with surface absorptance; the radiation absorbed by the back room is greater in December than it is in June; the radiation lost to the outdoors increases with decreasing opaque surface absorptance. These patterns appear in similar proportions for all three solutions; Figure 9 graphically shows the pattern comparison.

The closer the points are to the identity line, the closer the agreement among the simulations. The average distance from the identity line is 5.3±1.4 and 5.2±1.2 for the DEROB vs. the collapsed matrix and vs. TRNSYS data points, respectively. Note that maximum distance from the identity line is 70.7. Comparison of the data suggests that Wall may have exchanged the values for absorption between the sunroom and the back room for December, absorptivity = 0.2. The points in questions are highlighted in Figure 9.

CONCLUSIONS

In all cases, the rate of change of cavity absorptance of the attached room is slower compared to that of the sunspace when the surface absorptance changes. This is particularly true in the range from 0.2 to 0.9. This range encompasses most surfaces likely to be used in construction. The use of low-absorptance surfaces in the attached room makes it lighter; it is easier to illuminate. At the same time, the use of low-absorptance surfaces does not reduce significantly the cavity absorptance of the attached room. For clear glass, the cavity absorptance of the back room is greater in the wintertime, when solar gain is desirable. It is lower in the summertime when solar gain is unwanted. This is true, of course, provided the geometry of the spaces allows the solar beam to enter the attached space directly in the wintertime.

The solution for the distribution of diffuse radiation between two visually coupled spaces is given in closed form. I did the calculations for this paper in a spreadsheet program on a laptop computer; however, the method is sufficiently simple that we may even do the calculations manually. The closed form of the solution makes it an extremely fast routine for more complex programs used for building energy analysis.

The solution is physically correct, and it is firmly grounded on the principle of energy conservation. It is not an

TABLE 6
Radiation Lost to Outdoors, Absorbed by Back Room, and Absorbed by Sunspace During a Day in December and a Day in June*

Dec. abs = .8	Lost	Back Room	Sun Space	Dec. abs = .2	Lost	Back Room	Sun Space
Collapsed Matrix	14	17	69	Collapsed Matrix	52	20	28
DEROB	24	31	45	DEROB	52	28	20
TRNSYS	35	16	49	TRNSYS	47	17	36
Jun. abs = .8	**Lost**	**Back Room**	**Sun Space**	**Jun. abs = .2**	**Lost**	**Back Room**	**Sun Space**
Collapsed Matrix	24	12	63	Collapsed Matrix	60	16	26
DEROB	35	10	55	DEROB	68	11	21
TRNSYS	39	7	54	TRNSYS	53	9	38

* Opaque absorptance 80% and 20% for left and right tables. Values are expressed as percents of solar radiation that entered the sunspace.

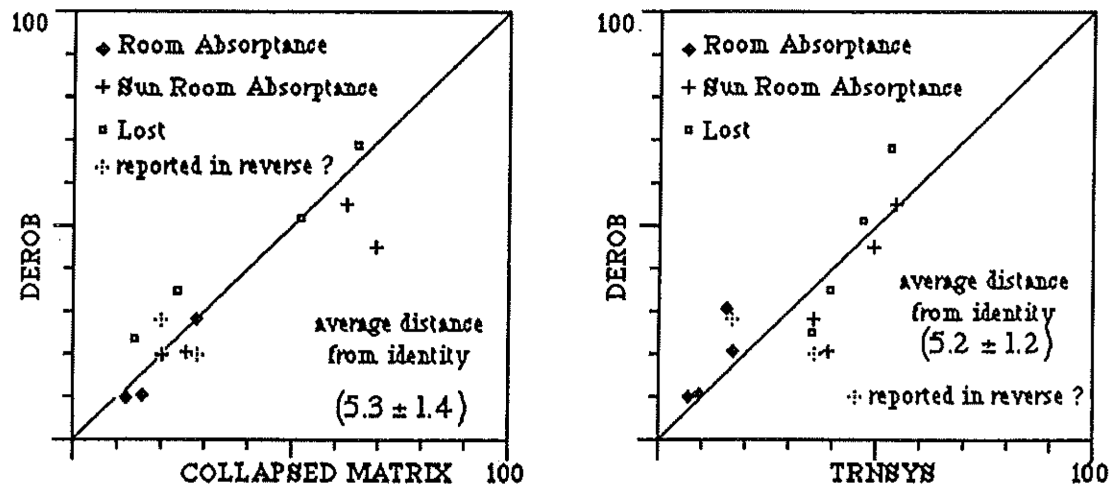

Figure 9 *Comparison of the TRNSYS and collapsed matrix solutions with the DEROB solution.*

approximation; it has collapsed information and as such it has limitations. It agrees with the DEROB calculations within 5.3%. Its very simplicity prevents it from providing detailed information on a surface-by-surface basis.

The method to solve for the infinite series as a manipulation on the statement of closure has general validity beyond its application to the collapsed matrix. It is the basis, in fact, for the radiation solution incorporated into DEROB.

NOMENCLATURE

A_i = area of surface i
a_i = absorptance coefficient of surface i
$\underline{\underline{a}}$ = diagonal matrix whose i component is a_i
α_i = total radiant power absorbed by surface i
$\underline{\alpha}$ = vector whose ith component is α_i
σ = denominator of converging infinite series
σ_1, σ_2 = elements in σ
f = area ratio
G_{ij} = form factor from surface j to surface i
$\underline{\underline{G}}$ = matrix whose $i\,j$ component is G_{ij}
J_{ij} = distributed form factor from surface j to surface i
$\underline{\underline{J}}$ = matrix whose $i\,j$ component is J_{ij}
$\underline{\underline{\mu}}$ = modifying matrix
R_i = total radiant power received by surface i
$\underline{R}$ = vector whose ith component is R_i
r_i = reflectance coefficient of surface i
$\underline{\underline{r}}$ = diagonal matrix whose i component is r_i
S_i = primary radiant power source from surface i
$\underline{S}$ = vector whose ith component is S_i
σ_{ij} = luminance factor from surface j to surface i
$\underline{\underline{\sigma}}$ = matrix whose $i\,j$ component is σ_{ij}
t_i = transmittance coefficient of surface i
$\underline{\underline{t}}$ = diagonal matrix whose i component is t_i
$\underline{\underline{\tau}}$ = outdoor components of
$\underline{u}$ = summation vector
ψ_i = total radiant power transmitted outdoors through surface i
$\underline{\psi}$ = vector whose ith component is ψ_i

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APPENDIX A

The distributed form factors for the collapsed matrix solution are

$$\underline{\underline{J}} = \frac{1}{\delta}\left(\begin{array}{c:c|c:c} f_1 r_2 \delta_2 & f_1 \delta_2 & t r_2 r_4 f_1 f_2 & t r_2 f_1 f_2 \\ \hdashline \delta_2 & [1 - f_1(1 - r_1)]\delta_2 + t^2 r_4 f_1 f_2 & t f_2 r_4 & t f_2 \\ \hline t r_2 r_4 f_1 f_2 & t r_4 f_1 f_2 & f_2 r_4 \delta_1 & f_2 \delta_1 \\ \hdashline t f_1 r_2 & t f_1 & \delta_1 & [1 - f_2(1 - r_3)]\delta_1 + t^2 r_2 f_1 f_2 \end{array}\right)$$

where

$$\delta = \delta_1 \delta_2 - (r_2 r_4)(f_1 f_2)t^2$$

$$\delta_1 = 1 - r_2[1 - (1 - r_1)f_1] \quad \delta_2 = 1 - r_4[1 - (1 - r_3)f_2]$$

The distributed form factors obey the symmetry condition $J_{ij}A_j = J_{ji}A_i$ and the closure condition $\underline{u} \cdot (\underline{\underline{a}} + \underline{\underline{\tau}}) \cdot \underline{\underline{J}} = \underline{u}$ where

$$\underline{\underline{a}} + \underline{\underline{\tau}} = \left(\begin{array}{c:c|c:c} a_1 & 0 & 0 & 0 \\ \hdashline 0 & a_2 + t_2 & 0 & 0 \\ \hline 0 & 0 & a_3 & 0 \\ \hdashline 0 & 0 & 0 & a_4 + t_4 \end{array}\right)$$

The matrix product gives the distributed form factor matrix, $\underline{\underline{J}} = \underline{\underline{G}} \cdot \underline{\underline{\sigma}}$

It is peculiar to the collapsed matrix solution that the distributed form factors are the same, independent of the clarity of the glazed partition that couples the two spaces. Clarity is the measure of the clearness vs. the translucence of the glass. Light transmitted through a translucent glass appears as a source of diffused light on the exit side of the glass. Light transmitted through a clear glass becomes a source only after it is reflected from a surface. The details of the calculation, however, are different for the two cases.

Clear Glass

$$\underline{\underline{G}} = \left(\begin{array}{c:c|c:c} 0 & f_1 & 0 & 0 \\ \hdashline 1 & (1 - f_1) & 0 & t f_2 \\ \hline 0 & 0 & 0 & f_2 \\ \hdashline 0 & t f_1 & 1 & (1 - f_2) \end{array}\right) \qquad \underline{\underline{r}} = \left(\begin{array}{c:c|c:c} r_1 & 0 & 0 & 0 \\ \hdashline 0 & r_2 & 0 & 0 \\ \hline 0 & 0 & r_3 & 0 \\ \hdashline 0 & 0 & 0 & r_4 \end{array}\right)$$

$$\underline{\underline{\sigma}} = \frac{1}{\delta}\left(\begin{array}{c:c|c:c} [1 - r_2(1 - f_1)]\delta_2 - r_2 r_4 t^2 f_1 f_2 & r_1 f_1 \delta_2 & t r_4 r_1 r_2 f_1 f_2 & t r_1 r_2 f_1 f_2 \\ \hdashline r_2 \delta_2 & \delta_2 & t f_2 r_4 r_2 & t f_2 r_2 \\ \hline t r_2 r_3 r_4 f_1 f_2 & t r_3 r_4 f_1 f_2 & [1 - r_4(1 - f_2)]\delta_1 - r_2 r_4 t^2 f_1 f_2 & r_3 f_2 \delta_1 \\ \hdashline t f_1 r_4 r_2 & t f_1 r_4 & r_4 \delta_1 & \delta_1 \end{array}\right),$$

$$\underline{\underline{\sigma}} = (\underline{\underline{1}} - \underline{\underline{r}} \cdot \underline{\underline{G}})^{-1}$$

Translucent Glass

$$\underline{\underline{G}} = \left(\begin{array}{c:c|c:c} 0 & f_1 & 0 & 0 \\ \hdashline 1 & (1 - f_1) & 0 & 0 \\ \hline 0 & 0 & 0 & f_2 \\ \hdashline 0 & 0 & 1 & (1 - f_2) \end{array}\right) \qquad \underline{\underline{r}} + \underline{\underline{\lambda}} = \left(\begin{array}{c:c|c:c} r_1 & 0 & t & 0 \\ \hdashline 0 & r_2 & 0 & 0 \\ \hline t & 0 & r_3 & 0 \\ \hdashline 0 & 0 & 0 & r_4 \end{array}\right)$$

$$\underline{\underline{\sigma}} = \frac{1}{\delta}\left(\begin{array}{c:c|c:c} [1 - r_2(1 - f_1)]\delta_2 & r_1 f_1 \delta_2 + f_1 f_2 r_4 t^2 & t f_2 r_4 [1 - r_2(1 - f_1)] & t f_2 [1 - r_2(1 - f_1)] \\ \hdashline r_2 \delta_2 & \delta_2 & t f_2 r_4 r_2 & t f_2 r_2 \\ \hline t f_1 r_2 [1 - r_4(1 - f_2)] & t f_1 [1 - r_4(1 - f_2)] & [1 - r_4(1 - f_2)]\delta_1 & r_3 f_2 \delta_1 + f_1 f_2 r_2 t^2 \\ \hdashline t f_1 r_4 r_2 & t f_1 r_4 & r_4 \delta_1 & \delta_1 \end{array}\right)$$

$$\underline{\underline{\sigma}} = [\underline{\underline{1}} - (\underline{\underline{r}} + \underline{\underline{\lambda}}) \cdot \underline{\underline{G}}]^{-1}$$

APPENDIX B

For a single uncoupled space, the collapsed matrix takes the form

$$\underline{\underline{J}} = \frac{1}{\delta}\left(\begin{array}{c:c} f r_2 & f \\ \hdashline 1 & [1 - f(1 - r_1)] \end{array}\right) \quad \delta = 1 - r_2[1 - f(1 - r_1)]$$

and for the special case when the space may be approximated with two large parallel surfaces, such as a solar collector or a Trombe wall,

$$f = 1 \quad \underline{\underline{J}} = \frac{1}{\delta}\left(\begin{array}{c:c} r_2 & 1 \\ \hdashline 1 & r_1 \end{array}\right) \quad \delta = 1 - r_1 r_2 \ .$$